

CKM and Tri-bimaximal MNS Matrices in a $SU(5) \times {}^{(d)}T$ Model

Mu-Chun Chen^{1,2,*} and K.T. Mahanthappa^{3,†}

¹*Department of Physics & Astronomy*

University of California, Irvine, CA 92697, USA

²*Theoretical Physics Department, Fermilab, Batavia, IL 60510-0500, USA*

³*Department of Physics, University of Colorado at Boulder, Boulder, CO 80309-0390, USA*

(Dated: February 1, 2008)

Abstract

We propose a model based on $SU(5) \times {}^{(d)}T$ which successfully gives rise to near tri-bimaximal leptonic mixing as well as realistic CKM matrix elements for the quarks. The Georgi-Jarlskog relations for three generations are also obtained. Due to the ${}^{(d)}T$ transformation property of the matter fields, the b -quark mass can be generated only when the ${}^{(d)}T$ symmetry is broken, giving a dynamical origin for the hierarchy between m_b and m_t . There are only nine operators allowed in the Yukawa sector up to at least mass dimension seven due to an additional $Z_{12} \times Z'_{12}$ symmetry, which also forbids, up to some high orders, operators that lead to proton decay. The resulting model has a total of nine parameters in the charged fermion and neutrino sectors, and hence is very predictive. In addition to the prediction for $\theta_{13} \simeq \theta_c/3\sqrt{2}$, the model gives rise to a sum rule, $\tan^2 \theta_\odot \simeq \tan^2 \theta_{\odot, \text{TBM}} - \frac{1}{2}\theta_c \cos \beta$, which is a consequence of the Georgi-Jarlskog relations in the quark sector. This deviation could account for the difference between the experimental best fit value for the solar mixing angle and the value predicted by the tri-bimaximal mixing matrix.

*mcchen@fnal.gov

†ktm@pizero.colorado.edu

I. INTRODUCTION

The measurements of neutrino oscillation parameters have entered a precision era. The global fit to current data from neutrino oscillation experiments give the following best fit values and 2σ limits for the mixing parameters [1],

$$\sin^2 \theta_{12} = 0.30 \text{ (} 0.25 - 0.34 \text{)}, \quad \sin^2 \theta_{23} = 0.5 \text{ (} 0.38 - 0.64 \text{)}, \quad \sin^2 \theta_{13} = 0 \text{ (} < 0.028 \text{)}. \quad (1)$$

These values for the mixing parameters are very close to the values arising from the so-called “tri-bimaximal” mixing (TBM) matrix [2],

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad (2)$$

which predicts $\sin^2 \theta_{\text{atm, TBM}} = 1/2$ and $\sin \theta_{13, \text{TBM}} = 0$. In addition, it predicts $\sin^2 \theta_{\odot, \text{TBM}} = 1/3$ for the solar mixing angle. Even though the predicted $\theta_{\odot, \text{TBM}}$ is currently still allowed by the experimental data at 2σ , as it is very close to the upper bound at the 2σ limit, it may be ruled out once more precise measurements are made in the upcoming experiments.

The finite group A_4 , which describes the even permutations of four objects and possesses four in-equivalent representations, 1, $1'$, $1''$ and 3, has been utilized as a family symmetry [3]. It was pointed out that the tri-bimaximal mixing matrix can arise when the A_4 family symmetry is imposed in the lepton sector [4]. However, due to its lack of doublet representations, CKM matrix is an identity in most A_4 models. In addition, to explain the mass hierarchy among the charged fermions, one needs to resort to additional symmetry. It is hence not easy to implement A_4 as a family symmetry for both quarks and leptons [5].

In this letter, we consider a different finite group, the double tetrahedral group, ${}^{(d)}T$, which is a double covering of A_4 . (For a classification of all finite groups up to order 32 that can potentially be a family symmetry, see [6]). Because it has the same four in-equivalent representations as in A_4 , the tri-bimaximal mixing pattern can be reproduced. In addition, ${}^{(d)}T$ has three in-equivalent doublets, 2, $2'$, and $2''$, which can be utilized to give the $2 + 1$ representation assignments for the quarks [7]. In the context of $SU(2)$ flavor group, this assignment has been known to give realistic quark mixing matrix and mass hierarchy [8]. Utilizing ${}^{(d)}T$ as a family symmetry for both quarks and leptons has been considered before in a $SU(5)$ model [9] and in a non-unified model [10]. In Ref. [9], both quarks and leptons (including the neutrinos) have $2 \oplus 1$ representation assignments under ${}^{(d)}T$, and the prediction for the solar mixing angle is $\sim 10^{-3}$, which is in the region of

	T_3	T_a	\overline{F}	H_5	H'_5	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	N	ξ	η
SU(5)	10	10	$\overline{5}$	5	$\overline{5}$	45	1	1	1	1	1	1	1	1
$^{(d)}T$	1	2	3	1	1	1'	3	3	2'	2	1''	1'	3	1
Z_{12}	ω^5	ω^2	ω^5	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1

TABLE I: Charge assignments. Here the parameter $\omega = e^{i\pi/6}$.

small mixing angle solution that has been ruled out by SNO and KamLAND. A recent attempt in [10] generalizes the $^{(d)}T$ to the quark sector while maintaining near TBM pattern. However, in order to explain the mass hierarchy, the model has to resort to an additional $U(1)$ symmetry. Furthermore, a large number of operators are present in this model, making it less predictive. Here we consider an SU(5) model combined with $^{(d)}T$ symmetry, which successfully accommodates the mass hierarchy as well as the mixing matrices in both quark and lepton sectors. With an additional $Z_{12} \times Z'_{12}$ symmetry, only “good” operators are allowed up to at least dimension seven, making the model very predictive. In addition, the mass hierarchy is naturally explained without having hierarchy in the vacuum expectation values (VEV’s) of the scalar fields, the reason being that the mass operators for the lighter generation are allowed to appear only at higher order compared to those for the heavy generation. Thus we have a dynamical explanation for the mass hierarchy.

II. THE MODEL

In SU(5), all matter fields are unified into a $10(Q, u^c, e^c)_L$ and a $\overline{5}(d^c, \ell)_L$ dimensional representations. The three generations of $\overline{5}$ are assigned into a triplet of $^{(d)}T$, in order to generate the tri-bimaximal mixing pattern in the lepton sector, and it is denoted by \overline{F} . On the other hand, to obtain realistic quark sector, the third generation of the 10-dim representation transforms as a singlet, so that the top quark mass is allowed by the family symmetry, while the first and the second generations form a doublet of $^{(d)}T$. These 10-dim representations are denoted by, respectively, T_3 and T_a , where $a = 1, 2$. The Yukawa interactions are mediated by a 5-dim Higgs, H_5 , a $\overline{5}$ -dim Higgs, H'_5 , as well as a 45-dim Higgs, Δ_{45} , which is required for the Georgi-Jarlskog relations. We have summarized these quantum number assignment in Table I. It is to be noted that H_5 and H'_5 are not conjugate of each other as they have different Z_{12} and Z'_{12} charges.

The Lagrangian of the model is given as follows,

$$\mathcal{L}_{\text{Yuk}} = \mathcal{L}_{\text{TT}} + \mathcal{L}_{\text{TF}} + \mathcal{L}_{\text{FF}} \quad (3)$$

$$\mathcal{L}_{\text{TT}} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi \zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3 \quad (4)$$

$$\mathcal{L}_{\text{TF}} = \frac{1}{\Lambda^2} y_b H_5' \bar{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \bar{F} T_a \phi \psi N + y_d H_5' \bar{F} T_a \phi^2 \psi' \right] \quad (5)$$

$$\mathcal{L}_{\text{FF}} = \frac{1}{M_x \Lambda} \left[\lambda_1 H_5 H_5 \bar{F} \bar{F} \xi + \lambda_2 H_5 H_5 \bar{F} \bar{F} \eta \right], \quad (6)$$

where M_x is the cutoff scale at which the lepton number violation operator $HH\bar{F}\bar{F}$ is generated, while Λ is the cutoff scale, above which the $^{(d)}T$ symmetry is exact. The parameters y 's and λ 's are the coupling constants. The vacuum expectation values (VEV's) of various SU(5) singlet scalar fields are,

$$^{(d)}T \longrightarrow G_{\text{TST}^2} : \quad \langle \xi \rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle \phi' \rangle = \phi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad (7)$$

$$^{(d)}T \longrightarrow G_{\text{T}} : \quad \langle \phi \rangle = \phi_0 \Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \psi \rangle = \psi_0 \Lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (8)$$

$$^{(d)}T \longrightarrow \text{nothing} : \quad \langle \psi' \rangle = \psi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (9)$$

$$^{(d)}T \longrightarrow G_{\text{S}} : \quad \langle \zeta \rangle = \zeta_0 \Lambda, \quad \langle N \rangle = N_0 \Lambda \quad (10)$$

$$^{(d)}T - \text{invariant} : \quad \langle \eta \rangle = u \Lambda \quad (11)$$

where G_{TST^2} denotes the subgroup generated by the elements TST^2 , which in the triplet representation is given by [10],

$$TST^2 = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad (12)$$

while G_{T} and G_{S} denote subgroup generated by the elements T and S , respectively. (Our notation is the same as in Ref. [10].) The details concerning vacuum alignment of these VEV's will be presented in a future publication.

We have summarized the remaining operators in the charged fermion sectors that are otherwise allowed by the $SU(5) \times ^{(d)}T$ symmetry in Table II. By imposing an additional $Z_{12} \times Z'_{12}$ symmetry,

under which the transformation properties of various fields are summarized in Table I, the above Lagrangian is the most general one. Here the operators that couple to $H_5 T_3 T_3$ are not shown in the above Lagrangian as their contributions can be absorbed into a redefinition of the coupling constant y_t . In addition, we neglect the operator $H'_5 \overline{F} T_3 \zeta \psi \psi'$ in \mathcal{L}_{TF} since its contribution is negligible. Also not shown are those that contribute to \mathcal{L}_{FF} which can be absorbed into a redefinition of the parameter u and ϕ_0 . Note that in principle, viable phenomenology may still be obtained when more operators are allowed. The additional discrete symmetry that is needed in that case would be smaller. Nevertheless, more Yukawa coupling constants will be present and the model would not be as predictive. The $Z_{12} \times Z'_{12}$ symmetry also forbids proton and other nucleon decay operators to very high orders; it is likely this symmetry might be linked to orbifold compactification in extra dimensions. Note that, the $Z_{12} \times Z'_{12}$ symmetry also separates the neutrino and charged fermion sectors, so that the neutrinos only couple to the G_{TST^2} breaking sector. Furthermore, it allows the 45-dim Higgs, Δ_{45} , to appear only in the operator shown above, and thus is crucial for obtaining the Georgi-Jarlskog (GJ) relations.

The interactions in \mathcal{L}_ν give the following neutrino mass matrix [3], which is invariant under G_{TST^2} [10],

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}, \quad (13)$$

and we have absorbed the Yukawa coupling constants by rescaling the VEV's. This mass matrix M_ν is form diagonalizable, *i.e.* the orthogonal matrix that diagonalizes it does not depend on the eigenvalues. Its diagonal form is,

$$V_\nu^T M_\nu V_\nu = \text{diag}(u + 3\xi_0, u, -u + 3\xi_0) \frac{v_u^2}{M_x}, \quad (14)$$

where the diagonalization matrix V_ν is the tri-bimaximal mixing matrix, $V_\nu = U_{\text{TBM}}$ given in Eq. 2. This tri-bimaximal mixing pattern and the mass eigenvalues in the neutrino sector are thus the same as in all previous analyses in models based on A_4 and $^{(d)}T$, which has been shown to be consistent with experimental data.

The down type quark and charged lepton masses are generated by \mathcal{L}_{TF} . Because the renormalizable operator $H'_5 \overline{F} T_3$ is forbidden by the $^{(d)}T$ symmetry, the generation of b quark mass requires the breaking of $^{(d)}T$, which naturally explains the hierarchy between m_t and m_b . The b quark mass, and thus the τ mass, is generated upon the breaking of $^{(d)}T \rightarrow G_{\text{T}}$ and $^{(d)}T \rightarrow G_{\text{S}}$. As m_b and m_τ

$H_5 T_3 T_a$	ψ', ψ $\psi\phi, \psi\phi', \psi'\phi, \psi'\phi', \psi'\zeta, \psi'N, \psi N$ $\psi^3, \psi\psi'^2, \psi\phi^2, \psi\phi'^2, \psi\phi\zeta, \psi\phi'\zeta, \psi'^3, \psi'\psi^2, \psi'\phi^2, \psi'\phi'^2, \psi'\phi\zeta, \psi'\phi'\zeta,$ $\psi\phi N, \psi\phi'N, \psi'\phi N, \psi'\phi'N$
	$\psi\xi, \psi'\xi, \psi\xi^2, \psi\xi\phi, \psi\xi\phi', \psi\xi\zeta, \psi'\xi^2, \psi'\xi\phi, \psi'\xi\phi', \psi'\xi\zeta, \psi\xi N, \psi'\xi N, \psi'\eta, \psi\phi\eta, \psi\phi'\eta, \psi\xi\eta,$ $\psi'\phi\eta, \psi'\phi'\eta, \psi'\xi\eta, \psi\eta, \psi\phi\eta, \psi\phi'\eta, \psi'\phi\eta, \psi'\phi'\eta, \psi\phi\eta, \psi\phi'\eta, \psi'\phi\eta, \psi'\phi'\eta$
$H_5 T_a T_a$	ϕ, ϕ' $\phi'^2, \psi^2, \psi'^2, \phi\phi', \psi\psi'$ $\phi^3, \phi^2\zeta, \phi\zeta^2, \phi'^2\zeta, \phi'\zeta^2, \phi\phi'\zeta, \phi\phi'^2, \phi'\phi^2, \phi N^2, \phi'N^2, \phi'^2N, \phi\phi'N, \phi N\zeta, \phi'N\zeta$
	$\xi, \xi^2, \xi\zeta, \xi N, \xi\eta, \xi^2, \xi\phi, \xi\phi', \xi^3, \xi^2\zeta, \xi^2\eta, \xi^2\zeta, \xi N\zeta, \xi N\eta, \xi\zeta\eta, \xi\phi^2, \xi\phi'^2, \xi\phi\phi',$ $\xi^2\phi, \xi^2\phi', \xi\phi N, \xi\phi\eta, \xi\phi\zeta, \xi\phi'N, \xi\phi'\eta, \xi\phi'\zeta, \phi^2\eta, \phi\eta^2, \phi\eta N, \phi\eta\zeta, \phi'\eta^2, \phi'\eta N,$ $\phi'\eta\zeta, \phi\eta, \phi'\eta, \xi N^2, \xi\eta^2, \xi\zeta^2$
$H_5' \overline{F} T_3$	ϕ, ϕ' $\psi^2, \phi^2, \phi'^2, \phi'\phi, \psi'^2, \psi\psi', \phi'\zeta, \phi'N, \phi N$ $\phi^3, \phi'^3, \phi^2\phi', \phi\phi'^2, \phi\zeta^2, \phi'\zeta^2, \phi\psi^2, \phi'\psi'^2, \zeta\psi^2, \zeta\psi'^2, \phi'\psi^2, \phi\psi^2,$ $\phi N^2, \phi'N^2, \phi N\zeta, \phi'N\zeta, N\psi^2, \zeta\psi^2, \zeta\psi\psi', N\psi\psi'$
	$\xi, \xi^2, \xi N, \xi\zeta, \xi\eta, \xi\phi, \xi\phi', \xi^3, \xi^2N, \xi^2\zeta, \xi^2\eta, \xi^2\phi, \xi^2\phi', \xi\phi^2,$ $\xi\phi'^2, \xi\phi\phi', \xi\phi N, \xi\phi\zeta, \xi\phi\eta, \xi\phi'N, \xi\phi'\zeta, \xi\phi'\eta, \phi'\eta, \phi\eta^2, \phi\eta N, \phi\eta\zeta, \phi'\eta^2, \phi'\eta N, \phi'\eta\zeta, \eta\psi^2,$ $\eta\psi'^2, \phi\eta, \phi\eta N, \phi\eta\zeta, \phi'\eta^2, \phi'\eta N, \eta\psi\psi'$
$H_5' \overline{F} T_a$	ψ, ψ' $\psi\phi', \psi'\phi, \psi'\phi', \phi\psi$ $\psi\phi^2, \psi\phi\zeta, \psi'\phi\zeta, \psi\phi'^2, \psi'\phi'^2, \psi\phi\phi', \psi'\phi\phi', \psi\phi'\zeta, \psi'\phi'\zeta, \psi\phi N, \psi'\phi N, \psi\phi'N, \psi'\phi'N$
	$\psi\xi, \psi'\xi, \psi\xi^2, \psi'\xi^2, \psi\xi\phi, \psi\xi\phi', \psi'\xi\phi, \psi'\xi\phi',$ $\psi\xi N, \psi\xi\eta, \psi\xi\zeta, \psi'\xi\zeta, \psi'\xi\eta, \psi'\xi N, \psi\phi\eta, \psi'\phi\eta, \psi'\phi'\eta, \psi\phi'\eta, \psi'\phi'\eta, \psi\phi\eta, \psi'\phi\eta$

TABLE II: Additional operators that are allowed by the $SU(5) \times {}^{(d)}T$ symmetry up to dim-7. For each operator shown above, there is a corresponding one with $H_5, H_5' \leftrightarrow \Delta_{45}$.

are generated by the same operator, $H_5' \overline{F} T_3 \phi\zeta$, we obtain the successful $b - \tau$ unification relation. Upon the breaking of ${}^{(d)}T \rightarrow G_T$, the operator $\Delta_{45} \overline{F} T_a \phi N$ contributes to the (22) element in $M_{d,e}$, and thus gives rise to m_s and m_μ . As this operator involves Δ_{45} , the GJ relation for the second family, $m_\mu \simeq 3m_s$ is obtained. If no further symmetry breaking takes place, the first generation masses, m_d and m_e vanishes. At this stage, the diagonalization mass matrix for the charged leptons (and down type quark) is identity, and hence the the tri-bimaximal mixing matrix is exact.

To obtain the correct mass relation for the first generation, it inevitably calls for flavor mixing

in the down quark sector, which then leads to corrections to the tri-bimaximal mixing pattern. The correction to the θ_{12} due to mixing in the charged lepton sector can account for the difference between $\sin^2 \theta_{12} = 1/3$ in the tri-bimaximal mixing matrix and the experimentally observed best fit value, $\sin^2 \theta_{12} = 0.3$. The GJ relation for the first family, $m_d \simeq 3m_e$, is obtained due to the operator $H'_5 \bar{F} T_a \phi^2 \psi'$, which further breaks the $^{(d)}T$ symmetry down to nothing. The mass matrices for the down type quarks and charged leptons are thus given by,

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi'_0 & 0 \\ -(1-i)\phi_0\psi'_0 & \psi_0 N_0 & 0 \\ \phi_0\psi'_0 & \phi_0\psi'_0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0, \quad (15)$$

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi'_0 & \phi_0\psi'_0 \\ (1+i)\phi_0\psi'_0 & -3\psi_0 N_0 & \phi_0\psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \quad (16)$$

where we have absorbed the coupling constants y_d and y_s by re-scaling the VEV's, ϕ_0 and ψ'_0 . Since the off diagonal elements in these mass matrices involve two VEV's, $\phi_0\psi'_0$, they are naturally smaller compared to ψ_0 , assuming the VEV's are naturally of the same order of magnitude. Besides explaining the mass hierarchy, it gives rise to the correct GJ relations in the first and the second families. Furthermore, as b is small, the corrections to θ_{12} and θ_{13} in the neutrino sector are under control. Note that there is no correction to $M_{d,e}$ given above at least to the order of dim-7.

The up quark masses are generated by the following Yukawa interactions, \mathcal{L}_{TT} . When the $^{(d)}T$ symmetry is exact, the only operator that is allowed is $H_5 T_3 T_3$, thus only top quark mass is generated, which naturally explains why the top mass is much larger than all other fermion masses. When $\langle \psi \rangle$ breaks $^{(d)}T$ down to G_T , the mass m_c and V_{td} is generated by the operators, $H_5 T_3 T_a \phi \zeta$ and $H_5 T_a T_a \phi^2$. The breaking of $^{(d)}T \rightarrow G_{TS^2}$ gives rise the up quark mass through the operator $H_5 T_a T_b \phi'^3$. These interactions give rise to the following mass matrix for the up type quarks,

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u, \quad (17)$$

where we have absorbed y_c/y_t and y_u/y_t by re-scaling the VEV's of ψ_0 and ϕ'_0 , and $y' = y_{ts}/\sqrt{y_c y_t}$.

The mixing angel θ_{12}^u from the up type quark mass matrix given in Eq. 17 is related to m_c and m_u as $\theta_{12}^u \simeq \sqrt{m_u/m_c}$, while the mixing angle θ_{12}^d arising from the down quark mass matrix M_d given in Eq. 15 is related to the ratio of m_d and m_s as $\theta_{12}^d \simeq \sqrt{m_d/m_s}$, to the leading order. The Cabibbo angle, θ_c , is therefore given by $\theta_c \simeq |\sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c}| \sim \sqrt{m_d/m_s}$, where the

relative phase α depends upon the coupling constants. Even though θ_{12}^d is of the size of the Cabibbo angle, the corresponding mixing angle in the charged lepton sector, θ_{12}^e , is much suppressed due to the GJ relations,

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3} \sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3} \theta_c. \quad (18)$$

As a result, the correction to the tri-bimaximal mixing pattern due to the mixing in the charged lepton sector is small, and is given, to the leading order, by,

$$\tan^2 \theta_\odot \simeq \tan^2 \theta_{\odot, \text{TBM}} - \frac{1}{2} \theta_c \cos \beta, \quad (19)$$

where the relative phase β is determined by the strengths and phases of the VEV's, ϕ_0 and ψ'_0 . Such a relation was also found in a model based on Pati-Salam gauge group [11] and is quite generic in models having Georgi-Jarlskog relations [12]. This deviation could account for the difference between the prediction of the TBM matrix, which gives $\tan^2 \theta_{\odot, \text{TBM}} = 1/2$, and the experimental best fit value, $\tan^2 \theta_{\odot, \text{exp}} = 0.429$, if $\cos \beta \simeq 2/3$ (with $\theta_c \simeq 0.22$). The off-diagonal matrix elements in M_e also generate a non-zero value for the neutrino mixing angle $\theta_{13} \simeq \theta_c/3\sqrt{2} \sim 0.05$. We note that a more precise measurement of $\tan \theta_\odot$ will pin down the phase of $\phi_0 \psi'_0$, and thus the three leptonic CP phases, which may yield interesting consequences on leptogenesis [13] and lepton flavor violating processes [14].

III. NUMERICAL RESULTS

The observed quark masses respect the following relation,

$$m_u : m_c : m_t = \epsilon_u^2 : \epsilon_u : 1, \quad m_d : m_s : m_b = \epsilon_d^2 : \epsilon_d : 1, \quad (20)$$

where $\epsilon_u \simeq (1/200) = 0.005$ and $\epsilon_d \simeq (1/20) = 0.05$.

In our model, the mass matrices for the down type quarks and charged leptons can be parametrized as,

$$\frac{M_d}{y_b v_d \phi_0 \zeta_0} = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & c & 0 \\ b & b & 1 \end{pmatrix}, \quad \frac{M_e}{y_b v_d \phi_0 \zeta_0} = \begin{pmatrix} 0 & -(1-i)b & b \\ (1+i)b & -3c & b \\ 0 & 0 & 1 \end{pmatrix}, \quad (21)$$

and with the choice of $b \equiv \phi_0 \psi'_0 / \zeta_0 = 0.00789$ and $c \equiv \psi_0 N_0 / \zeta_0 = -0.0474$, the mass ratios for the

down type quarks and for the charged leptons are given by,

$$m_d : m_s : m_b = 0.00250 : 0.0499 : 1.00 , \quad (22)$$

$$m_e : m_\mu : m_\tau = 0.000870 : 0.143 : 1.00 . \quad (23)$$

These predictions are consistent with the observed values given in Eq. 20 and are in good agreement with the GJ relations. The overall scale factor is $y_b \phi_0 \zeta_0 \simeq m_b/m_t \simeq (0.011)$ at the GUT scale, assuming the top Yukawa coupling is 1. For the up type quarks, the mass matrix can be written as,

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u , \quad (24)$$

and with the choice of $k \equiv y' \psi_0 \zeta_0 = -0.032$, $h \equiv \psi_0^2 = 0.0053$ and $g \equiv \phi_0'^3 = -2.25 \times 10^{-5}$, the ratio among the three up type quarks is given by,

$$m_u : m_c : m_t = 0.0000252 : 0.005 : 1.00 , \quad (25)$$

which is consistent with the observed values shown in Eq. 20. The absolute values of the CKM matrix elements are given by,

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.976 & 0.219 & 0.00780 \\ 0.219 & 0.975 & 0.0400 \\ 0.00638 & 0.0403 & 0.999 \end{pmatrix} . \quad (26)$$

Except for the element V_{ub} , which is slightly higher than the current experimental upper bound of ~ 0.005 , all other elements are in good agreement with current data. This discrepancy can be alleviated by allowing additional operators to be present in the model. It can also be improved by having complex parameters, with which realistic CP violation measures in the quark sector could also arise. We leave these possibilities for further investigation. The diagonalization matrix for the charged leptons is,

$$V_{e,L} = \begin{pmatrix} -0.996 + 0.052i & -0.0517 + 0.0581i & (6.35 - 6.36i) \times 10^{-5} \\ 0.0578 + 0.0520i & -0.995 + 0.0581i & 0.00108 - 0.0000636i \\ 7.24 \times 10^{-6} & 0.00109 & 0.999 \end{pmatrix} . \quad (27)$$

This leads to small deviation to the tri-bimaximal mixing pattern as discussed above, leading to

the following leptonic mixing matrix,

$$|U_{\text{MNS}}| = |V_{e,L}^\dagger U_{\text{TBM}}| = \begin{pmatrix} 0.838 & 0.545 & 0.0550 \\ 0.364 & 0.608 & 0.706 \\ 0.409 & 0.578 & 0.706 \end{pmatrix}, \quad (28)$$

which gives $\sin^2 \theta_{\text{atm}} = 1$, $\tan^2 \theta_{\odot} = 0.424$ and $|U_{e3}| = 0.055$. Note that the total number of parameters in our model is seven in the charged fermion sectors and two in the neutrino sector.

IV. CONCLUSION

In this letter, we have presented a grand unified model based on $\text{SU}(5)$ combined with the double tetrahedral group, $^{(d)}T$, which successfully, for the first time, gives rise to near tri-bimaximal leptonic mixing as well as realistic CKM matrix elements for the quarks. Due to the presence of the $Z_{12} \times Z'_{12}$ symmetry, only nine operators are allowed in the model, and hence the model is very predictive, the total number of parameters being nine in the Yukawa sector for the charged fermions and the neutrinos. In addition, it provides a dynamical origin for the mass hierarchy without invoking additional $\text{U}(1)$ symmetry. Due to the $^{(d)}T$ transformation property of the matter fields, the b -quark mass can be generated only when the $^{(d)}T$ symmetry is broken, which naturally explains the hierarchy between m_b and m_t . The $Z_{12} \times Z'_{12}$ symmetry, to a very high order, also forbids operators that lead to nucleon decays. In principle, a symmetry smaller than $Z_{12} \times Z'_{12}$ would suffice in getting realistic masses and mixing pattern; however, more operators will be allowed and the model would not be as predictive. We obtain the Georgi-Jarlskog relations for three generations. This inevitably requires non-vanishing mixing in the charged lepton sector, leading to correction to the tri-bimaximal mixing pattern. The model predicts non-vanishing θ_{13} , which is related to the Cabibbo angle as, $\theta_{13} \sim \theta_c/3\sqrt{2}$. In addition, it gives rise to a sum rule, $\tan^2 \theta_{\odot} \simeq \tan^2 \theta_{\odot, \text{TBM}} - \frac{1}{2}\theta_c \cos \beta$, which is a consequence of the Georgi-Jarlskog relations in the quark sector. This deviation could account for the difference between the experimental best fit value for the solar mixing angle and the value predicted by the tri-bimaximal mixing matrix.

Acknowledgments

We thank Carl Albright, Stefan Antusch and Chris Carone for useful comments. The work of M-CC was supported, in part, by the start-up funds from the University of California at Irvine. The work of KTM was supported, in part, by the Department of Energy under Grant no. DE-

FG02-04ER41290. M-CC thanks the Particle Theory Group at Fermilab for its hospitality while this work was performed.

-
- [1] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, New J. Phys. **6**, 122 (2004) [hep-ph/0405172v5, updated June 2006].
 - [2] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B530**, 167 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. **B535**, 163 (2002).
 - [3] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001); K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).
 - [4] E. Ma, Phys. Rev. **D70**, 031901 (2004).
 - [5] E. Ma, Mod. Phys. Lett. **A17**, 627 (2002); X. G. He, Y. Y. Keum and R. R. Volkas, JHEP **0604**, 039 (2006); E. Ma, H. Sawanaka and M. Tanimoto, Phys. Lett. **B641**, 301 (2006); I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. **B648**, 201 (2007); E. Ma, Mod. Phys. Lett. **A21**, 2931 (2006); S. F. King and M. Malinsky, Phys. Lett. **B645**, 351 (2007); S. Morisi, M. Picariello and E. Torrente-Lujan, Phys. Rev. **D75**, 075015 (2007).
 - [6] P. H. Frampton and T. W. Kephart, Int. J. Mod. Phys. **A10**, 4689 (1995); P. H. Frampton and T. W. Kephart, Phys. Rev. **D64**, 086007 (2001); see also, E. Ma, arXiv:0705.0327 [hep-ph].
 - [7] P. D. Carr and P. H. Frampton, hep-ph/0701034.
 - [8] See, *e.g.* M.-C. Chen and K. T. Mahanthappa, Phys. Rev. **D62**, 113007 (2000); *ibid.* **D65**, 053010 (2002); *ibid.* **D68**, 017301 (2003); for a review, see, *e.g.* Int. J. Mod. Phys. **A18**, 5819 (2003).
 - [9] A. Aranda, C. D. Carone and R. F. Lebed, Phys. Lett. **B474**, 170 (2000); A. Aranda, C. D. Carone and R. F. Lebed, Phys. Rev. **D62**, 016009 (2000).
 - [10] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, hep-ph/0702194.
 - [11] S. F. King, JHEP **0508**, 105 (2005).
 - [12] S. Antusch and S. F. King, Phys. Lett. **B631**, 42 (2005).
 - [13] See, *e.g.* M.-C. Chen and K. T. Mahanthappa, Phys. Rev. **D71**, 035001 (2005); *ibid.* **D75**, 015001 (2007); for a review, see *e.g.* M.-C. Chen, hep-ph/0703087 and references therein.
 - [14] See, *e.g.* M.-C. Chen and K. T. Mahanthappa, Phys. Rev. **D70**, 113013 (2004).